Tests of Universality of Baryon Form Factors in Holographic QCD

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We describe a new exact relation for large N_c QCD for the long-distance behavior of baryon form factors in the chiral limit, satisfied by all 4D semi-classical chiral soliton models. We use this relation to test the consistency of the structure of two different holographic models of baryons.

1. Introduction

There are still no systematic analytic tools to study the strong-coupling dynamics of QCD, except for models that probe certain limited classes of observables. Thus, for example, one can use chiral perturbation theory for some low-energy observables, but for more general ones one is forced to resort to more phenomenological approaches such as chiral soliton models. In recent years, holographic models of QCD have emerged as another approach to the low energy phenomenology of QCD, and have attracted considerable interest [1,2,3,4,5,6].

There are two distinct classes of AdS/QCD models. Top-down AdS/QCD models arise from string theory, with the D4/D8 system describing a gauge theory which is confining with non-Abelian chiral symmetry breaking as in QCD. Despite the fact that the classical limit of the AdS/CFT correspondence requires the number of colors and the 't Hooft coupling to be large [7], predictions of top-down AdS/QCD models, extrapolated to three colors, fare relatively well when compared to experimental data [8]. Botton-up model are also motivated by the AdS/CFT correspondence, but are more phenomenological and allow more freedom to match QCD data [1,2,4,5]. In these models, QCD in the large N_c limit is taken to be dual to a classical 5D theory in a curved space, and the parameters of the 5D model are matched to their corresponding values in large N_c QCD [9,10], with the field content of the 5D models chosen to match the low energy chiral symmetry of QCD. In contrast to approaches like chiral perturbation theory, these models allow the computation of meson spectra and couplings, at least in principle. Even very simple 5D models seem to show a remarkable phenomenological success when compared to data.

Given the many bold assumptions that are necessary to construct holographic models of QCD, their phenomenological success is remarkable, and may suggest that the assumptions are more reliable than might be expected. It is natural to wonder if there is anything in these models that can test the reliability of the assumptions. In this work we use a new model-independent relation for baryons, to test two new holographic models of baryons [11,12]. This relation is sensitive to the anomalous coupling of the baryon current to three pions and becomes an exact result of QCD in the combined large N_c and chiral limits (with the large N_c limit taken first).

In the large distance limit $r \to \infty$, the ratio of position-space electric and magnetic baryon form factors is given by [13]:

$$\lim_{r\to\infty} \frac{G_E^{I=0} G_E^{I=1}}{G_M^{I=0} G_M^{I=1}} = \frac{18}{r^2} \; . \tag{1}$$

These position-space form factors can be related to the standard experimentally accessible momentum-space form factors by

$$G_E^{I=0,1}(r) \quad = \quad \int \frac{d^3q}{(2\pi)^3}\; e^{i\vec{q}.\vec{r}} \tilde{G}_E^{I=0,1}(q)$$

$$G_M^{I=0,1}(r) = \frac{-i}{3}\int \frac{d^3q}{(2\pi)^3}\, e^{i\vec{q}.\vec{r}}\, \vec{q}.\vec{r}\, \tilde{G}_M^{I=0,1}(q) \quad (2)$$

2. 5D Skyrmions

The authors of refs. [11,14] use a simple holographic model for QCD, in which the Chern-Simon (CS) term is incorporated to take into account the QCD chiral anomaly. They show that the baryons arise as stable solitons which are the 5D analog of the 4D skyrmions.

The action of the 5D model is given by

$$\begin{aligned} S &= -\frac{M_5}{2}\int d^5x\sqrt{g}\ \mathrm{Tr}\ [\mathcal{L}_{MN}^2 + \mathcal{R}_{MN}^2] \\ &+ \frac{-iN_c}{24\pi^2}\int_{5D}[\omega_5(\mathcal{L}) - \omega_5(\mathcal{R})], \end{aligned} \quad (3)$$

where $\mathcal{L}_{MN}, \mathcal{R}_{MN}$ are the U(2) gauge field strengths, $M, N = z, \mu$, ω_5 is the Chern-Simons 5-form, and $M_5 \sim O(N_c^1)$ is an input parameter of the model.

In this model baryons are identified as the quantum states of slowly rotating '5D Skyrmions'. The 5D Skyrmions are defined to be topologically non-trivial configurations of the 5D gauge fields with baryon number $B = 1$. The hedgehog-like field configurations with $B = 1$ can be parametrized in terms of five functions $\phi_1(r,z)$, $\phi_2(r,z)$, $A_1(r,z)$, $A_2(r,z)$, $s(r,z)$. These functions satisfy some EOM, which are solved numerically.

By writing the isospin currents for the explicit case of a $B = 1$, slowly rotating 5D Skyrmion, it is possible to show that

$$\begin{aligned} G_E^{I=0}(r) &= -\frac{4}{N_c}M_5\left[\frac{a(z)\partial_z s}{r}\right]_{z=0}, \\ G_M^{I=0}(r) &= -\frac{2}{3N_c\mathcal{I}}M_5\,(ra(z)\partial_z Q\ )_{z=0}, \\ G_E^{I=1}(r) &= \frac{2}{3\mathcal{I}}M_5\,[a(z)\,(\partial_z v - 2(\partial_z\chi_2 \\ &- A_2\chi_1))]_{z=0}, \\ G_M^{I=1}(r) &= -\frac{4}{9}M_5\,[a(z)(\partial_z\phi_2 - A_2\phi_1)\ ]_{z=0} \end{aligned} \quad (4)$$

where $\mathcal{I}$ is the moment of inertia, and $v(r,z)$, $Q(r,z)$, $\chi_1(r,z)$, $\chi_2(r,z)$ parametrize the collective rotations of the 5D Skyrmion, and are defined in ref. [14].

The authors of ref. [14] also give the $r \to \infty$ behaviour of the functions $s(r,z)$, $Q(r,z)$, $v(r,z)$, $\chi_i(r,z)$ $\phi_i(r,z)$, $A_i(r,z)$, $i = 1,2$. These functions are parametrized in terms of the parameter β which is determined numerically. Using this we get for the large r limit of the form factors:

$$\begin{aligned} G_E^{I=0}(r\to\infty) &= -\frac{\beta^3L^6}{\pi^2}\frac{1}{r^9}, \\ G_M^{I=0}(r\to\infty) &= \frac{\beta^3L^6}{6\pi^2\lambda}\frac{1}{r^7}, \\ G_E^{I=1}(r\to\infty) &= \frac{8\beta^2}{3\lambda}M_5L^3\frac{1}{r^4}, \\ G_M^{I=1}(r\to\infty) &= -\frac{8\beta^2}{9}M_5L^3\frac{1}{r^4}, \end{aligned} \quad (5)$$

Using Eqs. (5) it is clear that Eq. (1) is satisfied in the Pomarol-Wulzer model of baryons as 5D Skyrmions. This means that the Pomarol-Wulzer model correctly captures the large N_c chiral physics of QCD to which Eq. (1) is sensitive.

3. Baryons as Holographic Instantons

In refs. [12,15], baryons are described as instantons in the 5D Yang-Mills (YM) and CS theory formulated in the D4/D8 model. It has been argued that the low energy phenomena of QCD can be derived from this model. In the case of the D4/D8 model, baryons are identified as D4-branes wrapped on a non-trivial four-cycle in the D4 background. Such a D4-brane is realised as a small instanton configuration in the world-volume gauge theory on the probe D8-brane. The action of the model is:

$$\begin{aligned} S &= -\kappa\int d^4x\ dz\ \mathrm{Tr}\left[\frac{1}{2}(1+z^2)^{-1/3}\mathcal{F}_{\mu\nu}^2\right. \\ &+ \left.(1+z^2)\mathcal{F}_{\mu z}^2\right] + \frac{N_c}{24\pi^2}\int\ \omega_5(\mathcal{A}), \end{aligned} \quad (6)$$

where $\mathcal{A}$ is the 5D $U(N_f)$ gauge field and $\mathcal{F}$ is the field strenght. The constant κ is related to the 't Hooft coupling, λ_t, and N_c by:

$$\kappa = \frac{\lambda_t N_c}{216\pi^3}. \quad (7)$$

and $\omega_5(\mathcal{A})$ is the CS 5-form for the $U(N_f)$ gauge field defined as

$$\omega_5(\mathcal{A}) = \text{ Tr } \left[\mathcal{A}\mathcal{F}^2 - \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right]. \quad (8)$$

Baryon in this model corresponds to a slowly rotating soliton with non-trivial instanton number on the four dimensional space parametrized by x^M $(M = 1, 2, 3, z)$.

To check whether this model satisfies Eq. (1), we use the expressions for the currents, derived in ref. [15] using the fact that the instantons are localized arbitrarily well at $z = 0$ in the large λ_t limit we get:

$$\begin{aligned}
G_E^{I=0}(r) &= -\sum_{n=1}^{\infty} g_{v^n}\psi_{2n-1}(0)Y_{2n-1}(r), \\
G_M^{I=0}(r) &= -\frac{9\pi r}{4\lambda N_c}\sum_{n=1}^{\infty} g_{v^n}\psi_{2n-1}(0) \\
&\times \quad m_{2n-1}Y_{2n-1}(r), \\
G_E^{I=1}(r) &= -\sum_{n=1}^{\infty} g_{v^n}\psi_{2n-1}(0)Y_{2n-1}(r), \\
G_M^{I=1}(r) &= -\frac{N_c}{3}\sqrt{\frac{2}{15}}\sum_{n=1}^{\infty} g_{v^n}\psi_{2n-1}(0) \\
&\times \quad \rho_{2n-1}Y_{2n-1}(r), \quad (9)
\end{aligned}$$

where $\{\psi_n(z)\}$ is a complete set of functions normalized so that $\psi(z) \sim \kappa^{-1/2}$ that satisfy $-(1+z)^{1/3}\partial_z(k(z)\partial_z\psi_n(z)) = \rho_n^2\psi_n(z)$, where the eigenvalues ρ_n^2 (with $\rho_{n+1} > \rho_n$) are related to the masses of the vector mesons in this model by $m_n^2 = \rho_n^2 M_{KK}^2$, the vector meson decay constants $g_{v^n} = 2\kappa \lim_{z\to\infty} z\psi_{2n-1}(z)$, and $Y_n(r)$ are Yukawa potentials $Y_n(r) = -e^{-\rho_n r}/(4\pi r)$.

From the expressions in Eqs. (9) one can see that $G_E^{I=0} = G_E^{I=1}$ and that the r dependence of $G_M^{I=0}$ is the same as $G_M^{I=1}$. This is very different from the r dependence obtained for these form factors in the Skyrme model, as shown in ref. [13]. As a matter of fact, the relation between $G_E^{I=0}$ and $G_E^{I=1}$ was noticed in ref. [15], since they got the same expression for the scalar and isovector charge distributions: $\rho_{I=1}(r) = \rho_{I=0}(r)$, although the isovector mean square radius should be divergent in the chiral limit. They argue that there is no contradiction in this result, since the divergence of the isovector mean square radius is due to the IR divergence of the pion loop, which is not included in their model. However, this result could be an indication that the chiral symmetry breaking is not correctly incorporated in their model and, in this case, we do not expect their form factors to satisfy the relation in Eq. (1). Besides, in the large r limit the scalar form factor is dominated by the 3-pion coupling with the nucleon, whereas the isovector form factor is dominated by the 2-pion coupling. Therefore, in the large r limit one expects a different r dependence for the scalar and isovector form factors.

Taking the large r limit of the form factors in Eqs. (9) we find that

$$\begin{aligned}
\lim_{r\to\infty} G_E^{I=0}(r) &= \frac{g_{v^1}\psi_1(0)}{4\pi r}e^{-\rho_1 r}, \quad (10) \\
\lim_{r\to\infty} G_M^{I=0}(r) &= \frac{9\pi r}{16\pi\lambda N_c}g_{v^1}\psi_1(0)\rho_1 e^{-\rho_1 r}, \\
\lim_{r\to\infty} G_E^{I=1}(r) &= \frac{g_{v^1}\psi_1(0)}{4\pi r}e^{-\rho_1 r}, \\
\lim_{r\to\infty} G_M^{I=1}(r) &= \frac{N_c}{12\pi}\sqrt{\frac{2}{15}}g_{v^1}\psi_1(0)\rho_1 \; e^{-\rho_1 r} ,
\end{aligned}$$

which makes it easy to see that

$$\lim_{r\to\infty}\frac{G_E^{I=0}G_E^{I=1}}{G_M^{I=0}G_M^{I=1}} = \frac{\lambda_t\sqrt{40/3}}{\pi\rho_1^2 r^2} . \quad (11)$$

As expected, the ratio is sensitive to model parameters, so Eq. (1) is not obeyed and the model does not correctly encode the large N_c chiral properties of baryons. This is troubling in that the ability to describe chiral symmetry and its spontaneous breaking are supposed to be principal virtues of the model. As mentioned above, the form factors depend only on couplings to vector mesons, and not to pions, in contradiction to large N_c χPT. This makes the failure of the model to satisfy Eq. (1) unsurprising. While the symptoms of the problem are clear, whether they represent a technical difficulty in the implementation of the model or a deeper structural problem remains an open question. It appears likely that the issue is connected to a non-commutativity of the large λ_t and chiral limits in this model.

4. Discussion

Our analysis above shows that the construction of the holographic model requires a number of ad hoc assumptions that are not always consistent with the low energy regime of QCD. The behavior of model independent relations, like the one in Eq. (1), is an explicit probe of the self-consistency of the assumptions. Clearly, if one wants to match the key features of large N_c QCD in a consistent way, it is essential to capture the scale dependence of QCD on the 5D side of the model. This amounts to trying to improve on the ad hoc approximations involved in the construction of the 5D model.

Although we have focused our analysis on the models of refs. [11,14,12,15], the problems in reproducing the low-energy regime of QCD apply rather broadly to holographic models of QCD.

Acknowledgements. A.C. and T.D.C. acknowledge the support of the US Dept. of Energy. M.N. acknowledges the support of CNPq and FAPESP.

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